AESB2320 Part 2 Exam Resit 2014

1. Since the filling is at uniform T at all times, we can do a macroscopic balance. We need first to determine the overall heat-transfer coefficient at the surface. Surface heat-transfer is like that in BSL1 Sect 9.6 (BSLK sect 10.3).

$$\frac{1}{r_0 U_0} = \left[\frac{1}{r_0 h_0} + \frac{\ln(r_1/r_0)}{k_1} + \frac{1}{r_1 h_1} \right]$$

$\frac{1}{r_0 h_0} \to$ 0, because we are given T at surface, not heat transfer coef.

$$\frac{1}{0.017\, U_0} = \left[\frac{\ln(0.019/0.017)}{0.2} + \frac{1}{(0.019)\,20} \right] = \left[0.556 + 2.63\right] = 3.19$$ (A)

$U_0 = 18.4\ W/(m^2 K)$

b. Macroscopic balance on the filling: (B)

$$\underbrace{(\pi R^2 L)\rho C_p \frac{dT}{dt}}_{\text{accum.}} = \underbrace{-U_0(2\pi R L)(T - T_{air})}_{\text{heat transfer at surface}}$$

$$\pi(0.017)^2(1000)(4190)\frac{dT}{dt} = 18.4 \cdot 2\pi(0.017)(T - T_a)$$

$$3804\, \frac{dT}{dt} = -1.96\,(T - T_a)$$

$$\frac{d(T-T_a)}{(T-T_a)} = 5.17 \cdot 10^{-4}\, dt \quad \text{. Integrate}$$

$$\ln(T - T_a) = 5.17 \cdot 10^{-4} + C_1 \quad \text{Use initial cond: at } t=0$$

$$\frac{T - T_a}{(T-T_a)_{t=0}} = \exp(-5.17\cdot 10^{-4} t)$$

$$\frac{50-20}{90-20} = \exp(-5.17 \cdot 10^{-4} t) \to t = 1640\ \text{sec} (\sim 27\ \text{min})$$

c. From Eq. 3, the resistance from convective heat transfer to the surface is 5 time larger from that for conduction through the crust. It is the more-important process (modes in series).

(The crust plays an important role not accounted for here: it slows or prevents evaporation from the surface, which would greatly accelerate convective heat transfer.)

2. This is unsteady conduction in a cylinder. Fig 11.5-2 (BSLK)
(Fig 12.1-2 in BSL2) applies.

We want the time until $\frac{T-T_0}{T_1-T_0} = \frac{50-90}{20-90} = 0.57$

My estimate from chart is $\frac{\alpha t}{R^2} \doteq 0.23$

$$\alpha = \frac{k}{\rho C_p} = \frac{0.68}{1000 \cdot 4190} = 1.62 \cdot 10^{-7}$$

$$\frac{1.62 \cdot 10^{-7}(t)}{(0.017)^2} = 0.23 \rightarrow t = 410\,s$$

3. These processes are in series. The slower estimate is better; answer 1, 1640 s.

[These questions were motivated by burning my mouth on a bitterbal, not a kroket. But we don't have the equations for question 1a in spherical geometry.]

4. Equation B in part 1b is of the form

$$(\text{Volume})\,\rho\, C_p \frac{dT}{dt} = -U_0\,(\text{Area})(T - T_{air})$$

Volume $= \pi R^2 L$ (again) $= \pi (0.017)^2 (0.08) = 7.26 \cdot 10^{-5}$

Area $= 2\pi RL + 2 \cdot \pi R^2 L$ - area increases bec. of the flat surfaces

Area $= 2\pi(0.017)(0.08) + \pi 2 (0.017)^2 = 0.00855 + 0.0018 = 0.0103$

The area term increases by a factor $\frac{0.0103}{0.00855} = 1.20$

The heat-transfer process would be 20% faster, and finish in $\frac{1640}{1.20} = 1370\,s$ (abt. 23 min.)

Note that Problem 1 assumes internal conduction is fast. The product method is not involved here.

5. Increasing convection always increases h. This is illustrated by the correlation between $Nu \equiv \frac{hD}{k}$ and Re for flow in pipes.

In this case, since convection of heat from the surface is the slow step in problem 1, and the process in problem 1 was slower than problem 2, we'd expect the whole cooling process to speed up.